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Analytical Kinematic analysis of Multi-DOF serial link robot arm

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Abstract: This paper proposes the technique to perform the forward and inverse kinematics solution for a multiple degree of freedom (dof) serial link manipulator. Forward kinematics is significant for any robotic arm as it helps to acquire the end effector position given any set of joint angles. Practically, however, the inverse kinematic equations are more widely used as just a set of coordinates of end effector position with respect to base frame are taken as input by which the joint angles are calculated. The forward kinematics study has been performed using the Denavit-Hartenberg method. The paper also discusses inverse kinematics using an analytical approach considering three joints of the robotic arm. After the study, a MATLAB Graphical User Interface has been designed in order to perform the forward and inverse kinematics along with the simulation of 6-dof serial link manipulator. This paper will be helpful to people who are trying to study and perform the kinematic analysis of a serial link manipulator from a beginner level.

Keywords: Robotic Arm, Forward Kinematics, Inverse Kinematics, Analytical Analysis, MATLAB

I. INTRODUCTION

Robotic technology has grown leaps and bounds in the past forty years [1]. Robots can exist in a variety of forms from a human like arm to a full human body type structure [2]. While a robot is designed, a proper kinematic study has to be carried out in order for it to operate properly. Kinematics is the study of the motion of an object without considering the forces that cause that motion.

The process through which we find the orientation and position of the end effector through the given joint angles is known as Forward Kinematics [3]. In a robotic arm, the joints can either revolute which provide rotary motion, or they can be prismatic, providing a linear motion. A robotic arm with three revolute joints is known as a RRR robot. Similarly, if it has two revolute and then a prismatic joint it is called a RRP Robot [4]. The revolute joints in the robotic arm are defined by a joint angle. Each joint connects two links, so the number of links is one greater than the number of joints. If we are given the joint angles, we can find the cumulative effect of the joint angles to find the pose of the end effector which means its orientation and position [5]. In industries, mostly six degree of freedom (dof) robot arms are used as their end effector position and orientation can be completely determined using forward kinematics [6]. Multi DOF robot arms especially serial link manipulators are widely used to perform repetitive pick and place tasks [7].

Kinematics study is important in order to design a robot arm as the kinematic model of a robot arm can be used to determine the motion planning and analyze changes caused due to the motion of the robot arm [8]. It also helps to define a relationship between all the joints and links of a robot arm for their position and velocity during motion [9].

The process by which joint angles of the actuators are calculated through the desired Cartesian position of the end effector is known as inverse kinematics. The geometric approach is the relatively easier choice when dealing with a two-dimensional (2D) arm as it’s convenient to visualize and calculations are simple. However, while dealing with three-dimensional (3D) arms, we generally find the link and joint parameters and the homogenous transformation matrices for all the joints to solve the forward kinematic problem. Lastly, the inverse kinematics problem is solved by making use of a mathematical approach [10].

II. MATHEMATICAL BACKGROUND

A. Forward Kinematics

Forward Kinematics is used to find the end effector coordinates when the joint angles of the actuators are provided [11]. Firstly, we are going to look at the forward kinematics of a 2 joint 2D robot arm. The End effector matrix will be given by:

$$E=R(q_1)Tx(a_1).....R(q_n)Tx(a_n) \tag{1}$$

Here n represents the number of joints in the robot, R and T represents the rotation matrix and translation matrix. The Rotation matrix for a 2D robot arm is given by (2).

$$R=\begin{bmatrix}\cos(q) & -\sin(q) & 0\\ \sin(q) & \cos(q) & 0\\ 0 & 0 & 1\end{bmatrix} \tag{2}$$

Here q is any arbitrary joint angle for robotic arm.

The translation matrix is given by (3) in which “a” is the link length from the base till the end effector.

$$T=\begin{bmatrix}1 & 0 & a\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \tag{3}$$

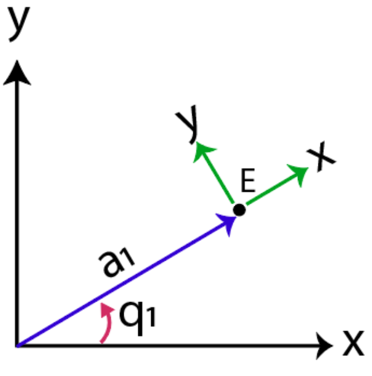


Fig. 1- Single Joint 2D.

For the given single joint arm, the joint is rotated by an angle q1 so the end effector pose will be given by:

$$E=\begin{bmatrix}cosq_1 & -sinq_1 & 0\\ sinq_1 & cosq_1 & 0\\ 0 & 0 & 1\end{bmatrix} * \begin{bmatrix}1 & 0 & a_1\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix} \tag{4}$$

$$E=\begin{bmatrix}cosq_1 & -sinq_1 & a_1cosq_1\\ sinq_1 & cosq_1 & a_1sinq_1\\ 0 & 0 & 1\end{bmatrix} \tag{5}$$

In (5), the $\begin{bmatrix}cosq_1 & -sinq_1\\ sinq_1 & cosq_1\end{bmatrix}$ component represents the orientation of the end effector. Whereas, $\begin{bmatrix}a_1cosq_1\\ a_1sinq_1\end{bmatrix}$ represents the x and y position of the end effector respectively.

Expanding it to a 2-joint 2-D arm

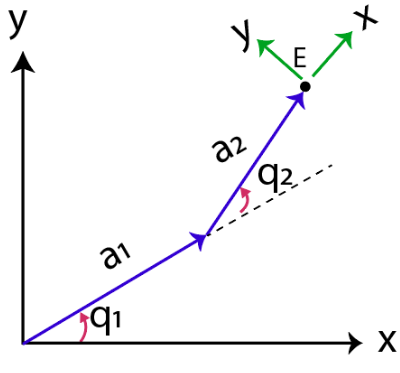


Fig. 2 - Two Joint 2D arm.

Here the end effector matrix will be given by

$$E = R(q_1)\ T_x(a_1)\ R(q_2)\ T_x(a_2) \tag{6}$$

This represents the rotations due to joint angles q_1 and q_2 and translations due to link lengths a_1 and a_2. Multiplying and simplifying (6) we get,

$$E=\begin{bmatrix}C_{12} & -S_{12} & a_2C_{12}+a_1C_1\\ S_{12} & C_{12} & a_2S_{12}+a_1S_1\\ 0 & 0 & 1\end{bmatrix}$$

Here

$C_1= \cos(q_1)$

$C_{12}= \cos(q_1+q_2)$

$S_1= \sin(q_1)$

$S_{12}= \sin(q_1+q_2)$

$\begin{bmatrix}a_2C_{12}+a_1C_1\\ a_2S_{12}+a_1S_1\end{bmatrix}$ represents the *x* and *y* positions of the end effector.

The forward kinematics for a multi-dof robot can be solved using the Denavit-Hartenberg convention. In this convention, each joint in the robotic arm is defined by 4 parameters: a_i which is the link length, α_i which is the link twist, d_i which is the link offset and q_i which is the joint angle [12]. Then the homogenous transformation of the joint is given by:

$$Ai= Rot_{z,qi}\ Trans_{z,di}\ Trans_{x,ai}\ Rot_{x,ai} \tag{7}$$

$$Ai=\begin{bmatrix}cq_i & -sq_ic\alpha_i & sq_is\alpha_i & a_icq_i\\ sq_i & cq_ic\alpha_i & -cq_is\alpha_i & a_isq_i\\ 0 & s\alpha_i & c\alpha_i & d_i\\ 0 & 0 & 0 & 1\end{bmatrix} \tag{8}$$

B. Inverse Kinematics

The process by which joint angles of the actuators are calculated using the end effector coordinates is called inverse kinematics. Deriving the inverse kinematic equations for 2 link planar robot arm:

From Fig.2 and (6), the end effector matrix will be given by:

$$E = R(q_1)\ T_x(a_1)\ R(q_2)\ T_x(a_2)$$

$$E=\begin{bmatrix}C_{12} & -S_{12} & a_2C_{12}+a_1C_1\\ S_{12} & C_{12} & a_2S_{12}+a_1S_1\\ 0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}x\\ y\end{bmatrix} = \begin{bmatrix}a_2C_{12}+a_1C_1\\ a_2S_{12}+a_1S_1\end{bmatrix} \tag{9}$$

Squaring and adding these two rows,

$$x^2+y^2 = (a_1)^2 + (a_2)^2 + 2a_1\, a_2\, C_2 \tag{10}$$

$$C_2=\frac{x^2+y^2-(a_1)^2-(a_2)^2}{2a_1\, a_2} \tag{11}$$

$$q_2 = \cos^{-1}\left(\frac{x^2+y^2-(a_1)^2-(a_2)^2}{2a_1\, a_2}\right) \tag{12}$$

Applying sum of angle identities on the rows from eq (9),

$$x = (a_1 + a_2C_2)C_1 - a_2S_2S_1 \tag{13}$$

$$y = (a_1 + a_2C_2)S_1 - a_2S_2C_1 \tag{14}$$

Using another identity,

$$a\cos(q) + b\sin(q) = C \tag{15}$$

where *a* and *b* are coefficients, and C is the result of the equation.

$$q = tan^{-1}\left(\frac{c}{\sqrt{(a^2+b^2-c^2)}}\right) - tan^{-1}\left(\frac{a}{b}\right) \tag{16}$$

In eq (16),

$a = a_2S_2$

$b = a_1 + a_2C_2$

$c = y$

$$q_1 = tan^{-1}\left(\frac{y}{\sqrt{((a_1)^2 + (a_2)^2 + 2a_1\, a_2\, C_2 - y^2)}}\right) - tan^{-1}\left(\frac{a_2S_2}{a_1 + a_2C_2}\right) \tag{17}$$

$$q_1 = tan^{-1}\left(\frac{y}{\sqrt{(x^2+y^2-y^2)}}\right) - tan^{-1}\left(\frac{a_2S_2}{a_1 + a_2C_2}\right) \tag{18}$$

$$q_1 = tan^{-1}\left(\frac{y}{x}\right) - tan^{-1}\left(\frac{a_2S_2}{a_1 + a_2C_2}\right) \tag{19}$$

So, we have got the answers for the joint angles q_1 and q_2 from (12) and (19):

$$q_1 = tan^{-1}\left(\frac{y}{x}\right) - tan^{-1}\left(\frac{a_2S_2}{a_1 + a_2C_2}\right)$$

$$q_2 = cos^{-1}\left(\frac{x^2 + y^2 - (a_1)^2 - (a_2)^2}{2a_1\, a_2}\right)$$

III. DESIGN AND IMPLEMENTATION OF MULTI-DOF ROBOT ARM

A. Forward Kinematic Equations

Our robotic arm consists of 5-joints (excluding the pose tip: end effector) and six links. The labelled image of our robotic arm is shown in Fig. 3 and Fig. 4 shows the robot at home position.

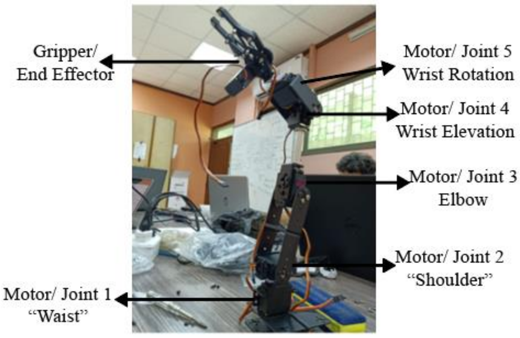


Fig. 3 Image of our 6 DOF Robotic Arm.

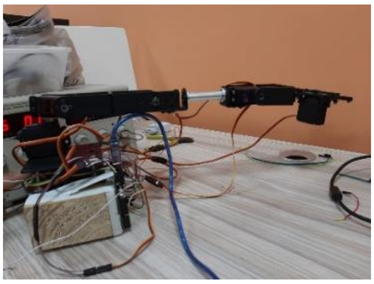


Fig. 4 Image of home position for our 6-DOF robotic arm.

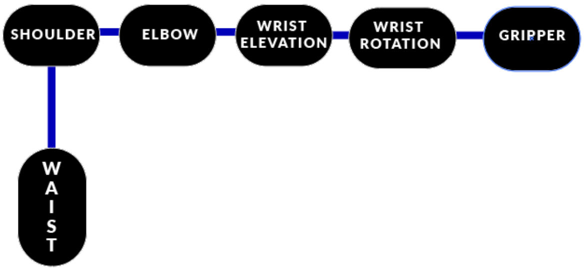


Fig. 5 Home position for our 6 DOF Robotic Arm.

Once, the home position is decided, now we have to define the axes for each of the joints. The axes are assigned to each joint keeping in mind DH convention rules.

Keeping in mind all the steps, procedure and rules, here are the coordinate frames for our robot:

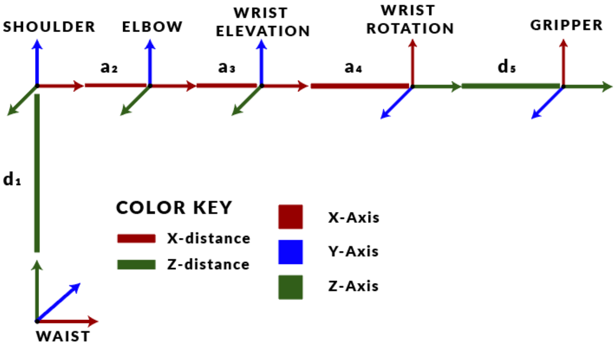


Fig. 6 Coordinate frames for our 6 DOF Robotic Arm.

The values for our link lengths are *d_1=1 cm, a_2=10.5 cm, a_3=14.7 cm, a_4=7.6 cm, d_5=11 cm.*

Once the coordinate frames are assigned, we measure the link twists and link lengths and then fill the DH in Table 1 where "i" indicates the joint number.

Table 1: DH notation for our 6-DOF Robotic Arm.

i	α_{i-1}	a_{i-1}	d_i	q_i
1	π/2	0	d_1	q_1
2	0	a_2	0	q_2
3	0	a_3	0	q_3
4	π/2	0	0	q_4+ π/2
5	0	0	a_4+ d_5	q_5

To find the pose of the end effector for a set of given joint angles, we have to first find the homogenous transformation matrices for each joint which is given by:

$$T = \begin{bmatrix} cq_i & -sq_i c\alpha_i & sq_i s\alpha_i & a_i cq_i \\ sq_i & cq_i c\alpha_i & -cq_i s\alpha_i & a_i sq_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{20}$$

After finding individual matrices for each joint, the matrices are multiplied together to find the end effector 4x4 matrix in which the (1,4), (2,4) and (3,4) values give the x, y and z positions of the end effector respectively.

B. Inverse Kinematic Equations

Now we will perform the inverse kinematics task for our 6-dof robot arm by using analytical approach. Keeping the third joint as redundant (positioned at its home position angle of 90 degrees) we will be finding the joint angles for the waist, shoulder and wrist elevation when we provide the robotic arm with the x, y and z coordinates of the end effector.

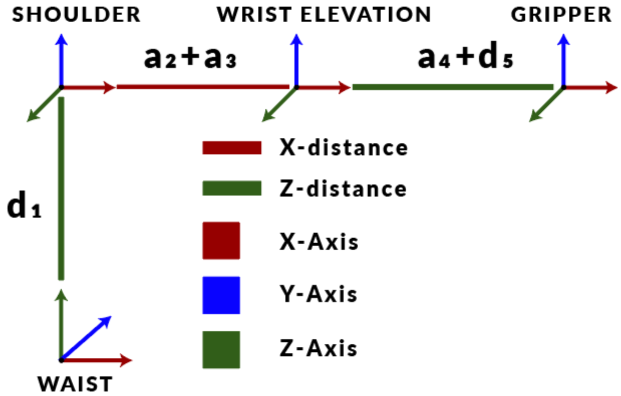


Fig. 7 Robot arm layout for Inverse Kinematics.

Here is the arm layout considering that we are using 3 motors for the end effector position i.e. Motor 1: Waist, Motor 2: Shoulder and Motor 4: Wrist Elevation, Motor 6 is simply used to open and close the gripper and hence it plays no role in the position of the end effector. Finding the angle for the waist, according to the axes assignment, it only changes the x and y position of the end effector and forms a circular path on the x-y plane hence it is very easy to find the angle associated with motor 1.

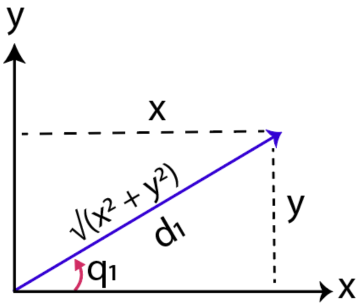


Fig. 8 Waist Angle.

The angle is

$$q_1 = tan^{-1}\frac{y}{x} \tag{21}$$

The subsequent motors can be drawn in a sideways view with the x-axis denoting the x-y plane that is the distance from the first motor till the projection of the end effector on the x-y plane.

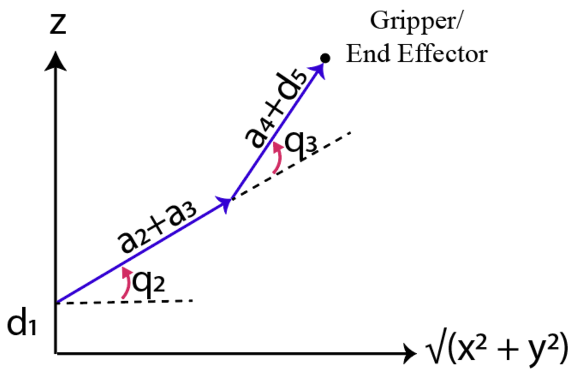


Fig. 9 Shoulder and Wrist elevation motors.

The derivation for these angles can be simply found out by our working of inverse on the 2-link planar robot as these two joints have similar axes orientation.

$$q_3 = cos^{-1}\left(\frac{x^2+y^2+(z-d_1)^2-(a_2+a_3)^2-(d_4+d_5)^2}{2(a_2+a_3)(d_4+d_5)}\right) \tag{22}$$

$$q_2 = tan^{-1}\left(\frac{(z-d1)}{\sqrt{(x^2+y^2)}}\right) - tan^{-1}\left(\frac{S_3(a4+d5)}{(a_2+a_3)+C_3(d_4+d_5)}\right) \tag{23}$$

IV. SIMULATION AND RESULTS

A. Forward Kinematics results

After performing the DH calculation, the forward kinematic equations came out to be:

X =

$$\frac{21C_{12}}{2} + \frac{93C_{124}S_3+C_{13}S_2}{5} + \frac{93C_{123}S_4+S_{23}C_1}{5} + \frac{147C_{123}}{10} - \frac{147S_{23}C_1}{10} \tag{24}$$

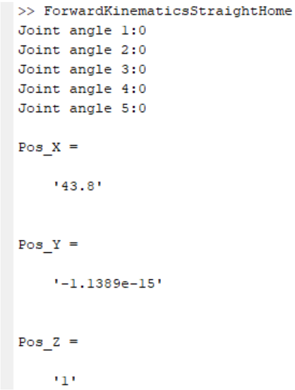
Y=

$$\frac{21S_1C_2}{2} + \frac{93C_{24}S_{13} + S_{12}C_3}{5} - \frac{93S_{1234} - C_{23}S_1}{5} - \frac{147S_{123}}{10} - \frac{147C_{23}S_1}{10} \tag{25}$$

Z=

$$\frac{21S_2}{2} + \frac{147S_3C_2}{10} + \frac{147S_2C_3}{10} - \frac{93C_{234} - S_2S_3}{5} + \frac{93S_{34}C_2 + S_2C_3}{5} + 1 \tag{26}$$

At the home position, all joint angles are zero, now we estimate the position of end effector at this home position. The position of end effector is always considered by the coordinate frame of the first joint. Hence, the x-position of the end effector should be $a_2+a_3+a_4+d_5 = 10.5 + 14.7 + 7.6 + 11 = 43.8$ *cm*. There is no y-displacement, so the y-position is 0 with respect to the base coordinate frame. The z-position should be $d_1 = 1$ *cm*. So, the *x, y, z* positions are *(43.8, 0, 1)*. Now we need to verify this position by the forward kinematic equations by putting all joint angles as zero degrees. This was verified on MATLAB along with a Graphical User Interface (GUI) design to input joint angles and get the end effector positions.



```
>> ForwardKinematicsStraightHome
Joint angle 1:0
Joint angle 2:0
Joint angle 3:0
Joint angle 4:0
Joint angle 5:0

Pos_X =

    '43.8'


Pos_Y =

    '-1.1389e-15'


Pos_Z =

    '1'
```

Fig. 10 MATLAB result verifying our Forward kinematic equations.

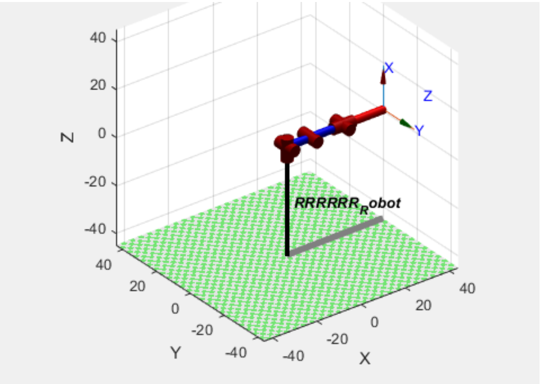
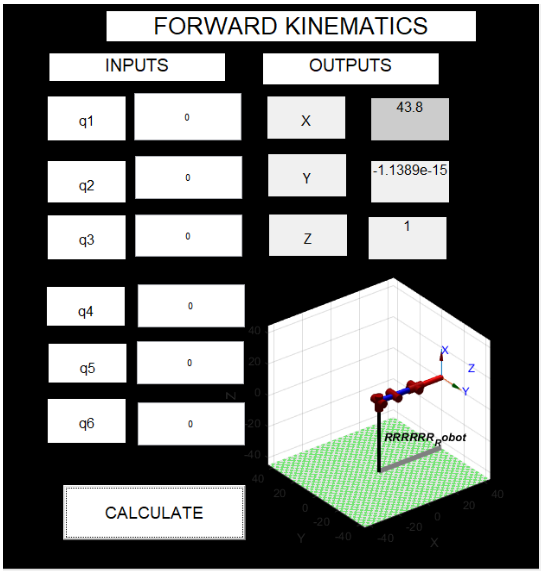


Fig. 11 Our 6R robotic arm simulated in MATLAB.



FORWARD KINEMATICS

INPUTS

q1 0
q2 0
q3 0
q4 0
q5 0
q6 0

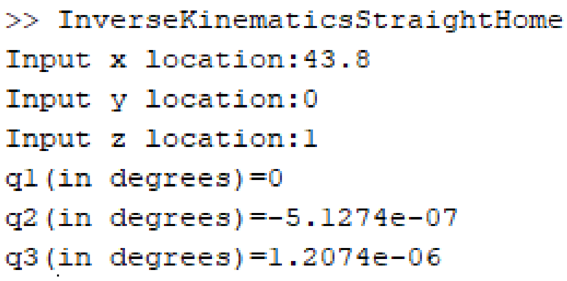
OUTPUTS

X 43.8
Y -1.1389e-15
Z 1

Fig. 12 MATLAB Graphical User Interface (GUI) for forward kinematics.

B. Inverse Kinematics results

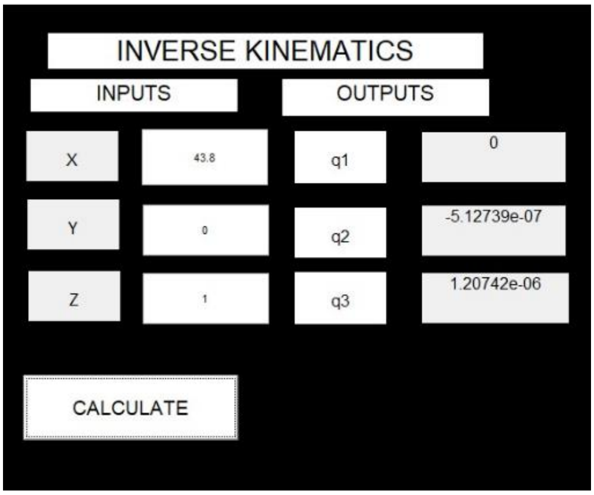
In order to find out the joint angles from a given position, the home position will be used in which the motor angles are zero, the x-position of end effector is $a_2+a_3+a_4+d_5= 43.8$ *cm*, the y-position is zero and z-position is $d_1=1cm$. So, the end effector position $(x, y, z) = (43.8, 0, 1)$. Now to verify if the inverse kinematic equations are valid, we need to give the equations this position and verify if the joint angles q_1, q_2 and q_3 come out to be zero. A code is developed in MATLAB using the q_1, q_2, q_3 equations that were derived in the last chapter. The code is shown below along with the correct output that was needed, i.e. all joint angles are zero.



```
>> InverseKinematicsStraightHome
Input x location:43.8
Input y location:0
Input z location:1
q1(in degrees)=0
q2(in degrees)=-5.1274e-07
q3(in degrees)=1.2074e-06
```

Fig. 13 MATLAB Code to verify inverse kinematic equations.

After this verification on the home position, verification for a couple of different positions was carried out and again the correct, desired angles were achieved. Thus, the inverse kinematics was verified using three joint angles. The gripper can also be oriented using motor 5 (wrist rotation) if another input in the form of orientation matrix is provided. Graphical User Interface (GUI) for inverse kinematics.



INVERSE KINEMATICS

INPUTS

X 43.8
Y 0
Z 1

OUTPUTS

q1 0
q2 -5.12739e-07
q3 1.20742e-06

Fig. 14 MATLAB GUI for inverse Kinematics.

The MATLAB code and files for the designed Graphical User Interface to perform the Forward and Inverse Kinematic calculations can be found on [https://www.mathworks.com/matlabcentral/fileexchange/89684-gui-of-forward-and-inverse-kinematics-for-multi-dof-robot.](https://www.mathworks.com/matlabcentral/fileexchange/89684-gui-of-forward-and-inverse-kinematics-for-multi-dof-robot)

V. CONCLUSION

This paper discusses the mathematical implementation of kinematics along with simulation of a multi-DOF robot arm. Moving forward, a vehicle will be attached below the robot arm changing it to a mobile robot, dynamic analysis will be done, and pick and place tasks will be conducted using the mobile robot.

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